Hybrid and Timed Systems

modeling, theory, verification

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Revised for MPRI course 2-8-2
2022-23
(barred slides are not in my programme this year)

Introductory equations

Hybrid Systems

- **Hybrid Systems** = Discrete+Continuous
- **Hybrid Automata** = A model of Hybrid systems
- **Original motivation**= Physical plant + Digital controller
- **New applications** = biology, economy, numerics, circuits
- **Hybrid community** = Control scientists + Applied mathematicians + Some computer scientists

Cyber-Physical!

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Cyber-Physical

Timed Systems

and a nice automata/ languages theory

- **Timed Systems** = Discrete behavior+Continuous Time
- **Timed Automata** = A subclass of Hybrid automata
- **The starting point** = A beautiful result by Alur & Dill.
- **Applications**= Real-time digital system, etc...
- **Timed community** = Computer scientists

Global Outline

1. Hybrid Automata (see Laurent Fribourg's lectures)
2. Timed Automata
3. ~~Back to Hybrid: Decidable Subclasses~~

Hybrid automata: the model
An example
Definition of HA
Classes of HA
A couple of exercises
Verification of HA
The reachability problem
The curse of undecidability
How to verify HA: theory and practice

Part I

Hybrid Automata

Outline

1. Hybrid automata: the model
   - An example
   - Definition of HA
   - ~~Classes of HA~~
   - A couple of exercises
2. Verification of HA
   - The reachability problem
   - The curse of undecidability
   - How to verify HA: theory and practice

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The first (cyber-physical) example

Notation

For $x = x(t)$ we write $\dot{x} = \dot{x}(t) = x'(t) = dx/dt$.

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Notation

For $x = x(t)$ we write $\dot{x} = \dot{x}(t) = x'(t) = dx/dt$.

[redacted]

- When the heater is OFF, the room cools down :

$$\dot{x} = -x$$

- When it is ON, the room heats:

$$\dot{x} = H - x$$

The first (cyber-physical) example

Notation

For $x = x(t)$ we write $\dot{x} = \dot{x}(t) = x'(t) = dx/dt$.

A thermostat

- When the heater is OFF, the room cools down :

$$\dot{x} = -x$$

- When it is ON, the room heats:

$$\dot{x} = H - x$$

X

- When [redacted] $> M$ it switches OFF
- When [redacted] $< m$ it switches ON

X

The first (cyber-physical) example

Notation

For $x = x(t)$ we write $\dot{x} = \dot{x}(t) = x'(t) = dx/dt$.

A thermostat

- When the heater is OFF, the room cools down :

$$\dot{x} = -x$$

- When it is ON, the room heats:

$$\dot{x} = H - x$$

- When $t > M$ it switches OFF
- When $t < m$ it switches ON

A strange creature...

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A bad syntax

Some mathematicians prefer to write

$$\dot{x} = f(x, q)$$

where

$$\begin{aligned} f(x, \mathrm{Off}) &= -x \\ f(x, \mathrm{On}) &= H - x \end{aligned}$$

with some switching rules on q.

A bad syntax

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where

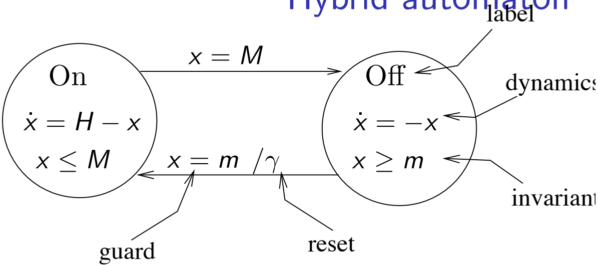
$$\begin{aligned} f(x, \mathrm{Off}) &= -x \\ f(x, \mathrm{On}) &= H - x \end{aligned}$$

with some switching rules on q.

But we are computer scientists

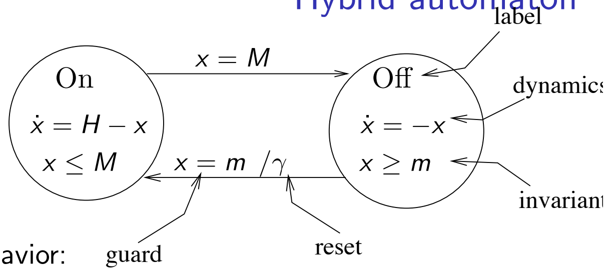
and draw an *automaton*

Hybrid automaton

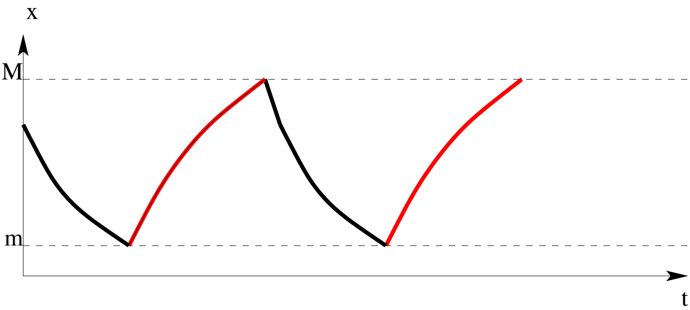


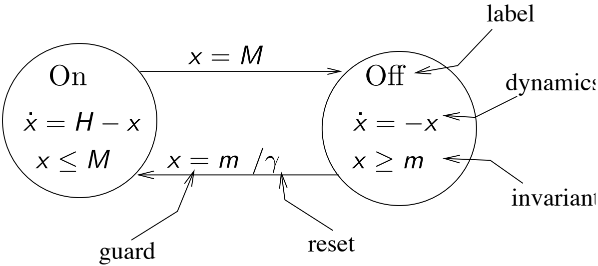
A formal definition: It is a tuple . . .

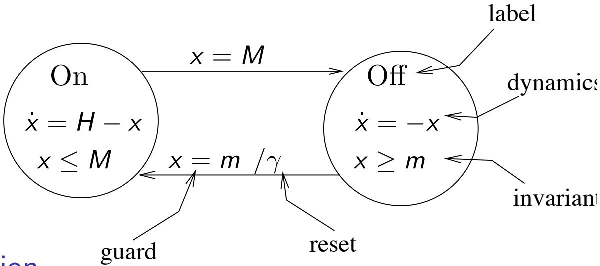
Hybrid automaton



Its behavior:



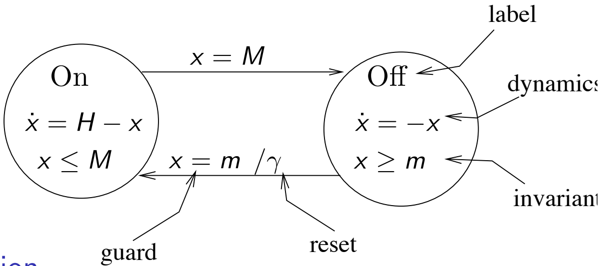

label
dynamics
invariant
guard
reset
On
$\dot{x} = H - x$
$x \leq M$
Off
$\dot{x} = -x$
$x \geq m$
$x = M$
$x = m \ /\gamma$



Definition

A hybrid automaton is $H = (Q, X, \Sigma, Dyn, I, \Delta)$ with

- Q finite set of locations
- $X = \mathbb{R}^n$, continuous state space
- Dyn, dynamics on X for every $q \in Q$
- I, invariant, staying condition in X
- Δ, finite set of transitions $\delta = (p, q, a, g, r)$



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- $X = \mathbb{R}^n$, continuous state space , a point in X = valuation of continuous variables $\mathbf{x} = x_1, \dots, x_n$
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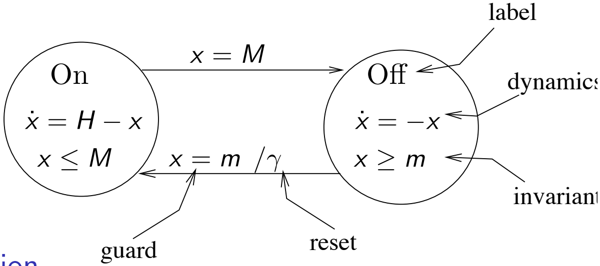

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- $X = \mathbb{R}^n$, continuous state space
- *Dyn*, dynamics on X for every $q \in Q$, $Dyn(q) = f_q$, whenever in location q the continuous state obeys $\dot{\mathbf{x}} = f_q(\mathbf{x})$.
- I, invariant, staying condition in X
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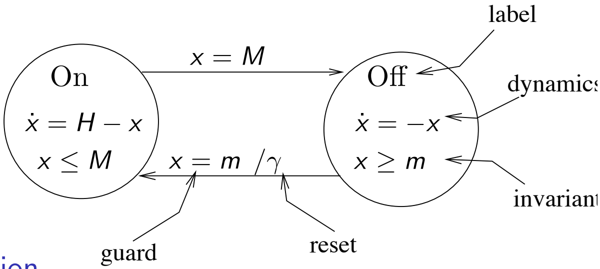

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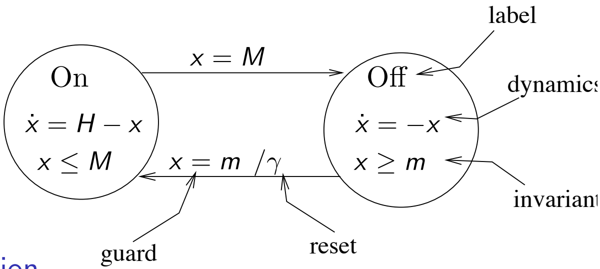

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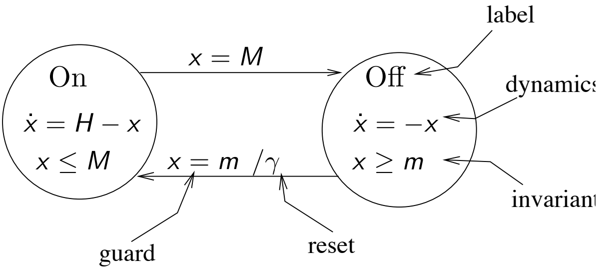
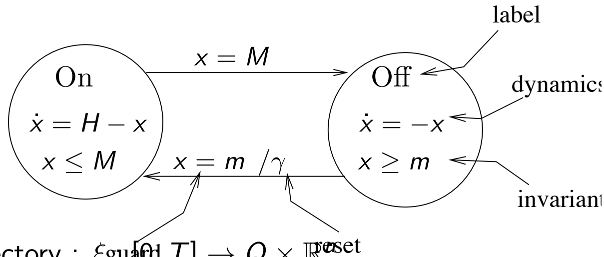
- Δ, finite set of transitions $\delta = (p, q, a, g, r)$
  - $p, q \in Q$, from p to q
  - $a \in \Sigma$ a label
  - g a guard; $g(\mathbf{x})$ required to take δ
  - r a reset (or jump); $\mathbf{x} := r(\mathbf{x})$ when taking δ

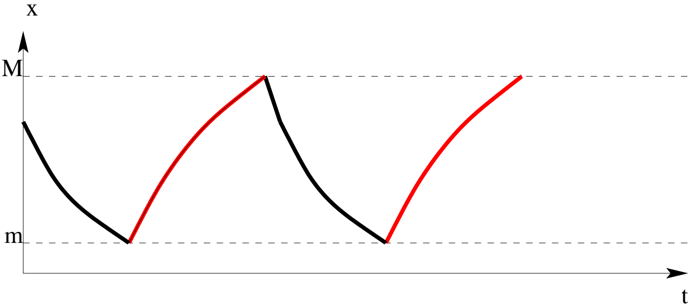
Trajectory-based semantics



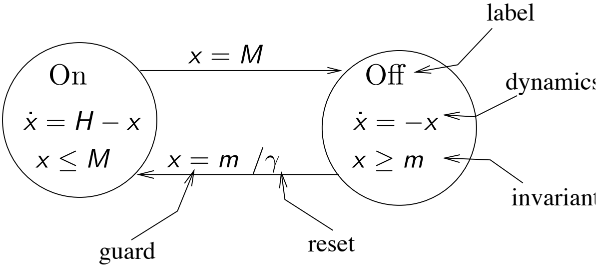
Trajectory-based semantics



A trajectory : $\xi : [0, T] \to Q \times \mathbb{R}^n$



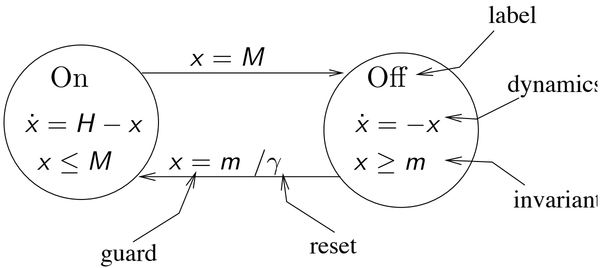
Transition system semantics



Transition system (S, T) of a HA

- **States:** $S = Q \times \mathbb{R}^n$
- **Transitions:** $T = T_{\text{flow}} \cup T_{\text{jump}}$

Transition system semantics



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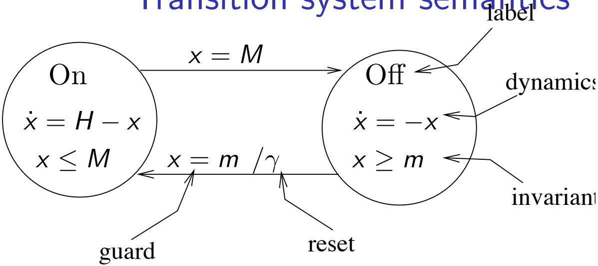
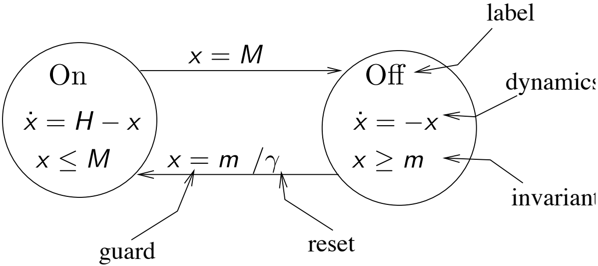


Transition system (S, T) of a HA

- **States:** $S = Q \times \mathbb{R}^n$
- **Transitions:** $T = T_{\text{flow}} \cup T_{\text{jump}}$
    - $(q, \mathbf{x}_1) \overset{\text{flow}}{\to} (q, \mathbf{x}_2) \Leftrightarrow$ we can go from $\mathbf{x}_1$ to $\mathbf{x}_2$ in ODE $\dot{\mathbf{x}} = f_q(\mathbf{x})$
    - $(q_1, \mathbf{x}_1) \overset{\text{jump}}{\to} (q_2, \mathbf{x}_2) \Leftrightarrow$ if we can jump.

Transition system semantics



Transition system (S, T) of a HA

- **States:** $S = Q \times \mathbb{R}^n$
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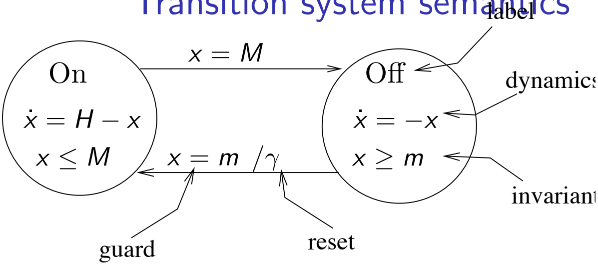
- **Runs:** sequences of states and transitions.

Transition system semantics



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- **Runs:** sequences of states and transitions.

$$(\mathrm{On}, 0) \xrightarrow{\text{flow}} (\mathrm{On}, M) \xrightarrow{\text{jump}} (\mathrm{Off}, M) \xrightarrow{\text{flow}} (\mathrm{Off}, m) \xrightarrow{\text{jump}} (\mathrm{On}, m) \cdots$$

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Classes of Hybrid Automata

Why classes?

Because HA are too reach; it is impossible to establish, decide, analyze properties of all HA.

How to define a class of HA

- dimension, discrete or continuous time, eager or lazy
- what kind of dynamics
- what kind of guards/invarians/jumps

We will consider TIMED AUTOMATA

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How to model?

Different systems

- a control system
- a scheduler with preemption
- a genetic network

The same class of models

A network of interacting Hybrid automata

Modeling exercise 1

Genetic network

We consider expression of two genes A and B, i.e. production of two proteins P and Q

- The proteins are degraded with rate k.
- P catalyzes expression of B:
  - Production of Q is proportional to the concentration of P with a coefficient a.
  - Concentration of P crosses a threshold $s \Rightarrow$ production of Q constant $= as$.
- Q inhibits expression of A:
  - Production of P equals $d - b \cdot$(concentration de Q).
  - Concentration of Q crosses a threshold $r \Rightarrow$ production of P blocks.

Modeling exercise 2

Scheduling

Schedule two jobs on one CPU and one printer with a total execution time up to 16 minutes.

- Job 1 : Compute (10 min); Print (5 min)
- Job 2 : Download (3 min); Compute (1 min); Print (2 min)

Try it :

1. without preemption;
2. with preemptible computing.

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Verification and reachability problems

- Is automatic verification possible for HA?

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Verification and reachability problems

- Is automatic verification possible for HA?
- *Safety:* are we sure that HA never enters a bad state?
- It can be seen as reachability : verify that

$$\neg \mathrm{Reach}(\mathit{Init}, \mathit{Bad})$$

Verification and reachability problems

- Is automatic verification possible for HA?
- *Safety:* are we sure that HA never enters a bad state?
- It can be seen as reachability : verify that

$$\neg\mathrm{Reach}(\mathit{Init}, \mathit{Bad})$$

- It is a natural and challenging mathematical problem.
- Many works on decidability
- Some works on approximated techniques

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The reachability problem for a class C

Problem

Given

- *a hybrid automaton* $\mathcal{H} \in C$
- *two sets* $A, B \subset Q \times \mathbb{R}^n$

find out whether there exists a trajectory of $\mathcal{H}$ *starting in* A *and arriving to* B.

All parameters rational.

Exact methods: The curse of undecidability

Bad news

- Koiran et al.: Reach is undecidable for 2d PAM.
- AM95: Reach is undecidable for 3d PCD.
- HPKV95 Many results of the type : "3clocks + 2 stopwatches = undecidable"

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They are really bad

- Reachability is undecidable for very simple HA.
- Thus, other verification problems are also undecidable.

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Undecidability Proofs — Preliminaries

Proof method:

simulation of Minsky Machine, Turing Machine etc.

Undecidability Proofs — Preliminaries

Proof method:

simulation of Minsky Machine, Turing Machine etc.

Details: proof schema

- Reachability undecidable for Minsky Machines (well-known).
- A class of HA can simulate MM (to prove).
- Reach for MM $\preceq$ Reach for HA.
- Conclude that Reach for HA is undecidable.

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Minsky Machines

Definition

- A counter: values in $\mathbb{N}$; operations: $C++$, $C--$; test $C > 0$?
- A Minsky machine has 2 counters
- Its program has finitely many lines like that:

$$\begin{array}{lll} q_1: & D++; & \text{goto } q_2 \\ q_2: & C--; & \text{goto } q_3 \\ q_3: & \text{if } C>0 & \text{then goto } q_2 \text{ else } q_1 \end{array}$$

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Theorem (Minsky)

Reachability is undecidable for Minsky machines.

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Theorem (Minsky)

(All variants: (p,0,0)->(q,0,0); (p,0,0)->(q,*,*); or (p,n,0)-> (q,*,*) even for a fixed machine, etc

Reachability is undecidable for Minsky machines.

Fact

Any algorithm can be programmed on a Minsky machine. But they are slooooooow.

A typical undecidability theorem

Theorem (Koiran, Cosnard, Garzon)

Reach *is undecidable for 2d PAM.*

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A typical undecidability theorem

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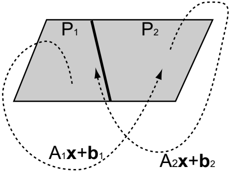
Reminder

A 2 dimensional PAM:



$\mathbf{x} := A_i\mathbf{x} + \mathbf{b}_i$ for $\mathbf{x} \in P_i$

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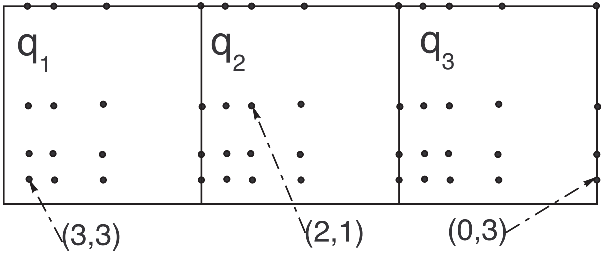
Simulating a counter by a PAM

Counter	PAM
State space $\mathbb{N}$	State space $[0;1]$
State $C = n$	$x = 2^{-n}$
$C++$	$x := x/2$
$C--$	$x := 2x$
$C > 0?$	$x < 0.75?$

Encoding a state of a Minsky Machine



Minsky Machine	**PAM**
State space $\{q_1, \dots, q_k\} \times \mathbb{N} \times \mathbb{N}$	State space $[1; k+1] \times [0; 1]$
State $(q_i, C = m, D = n)$	$x = i + 2^{-m}, y = 2^{-n}$

Simulating a Minsky Machine

Minsky Machine	**PAM**
State space $\{q_1, \dots, q_k\} \times \mathbb{N} \times \mathbb{N}$	State space $[1; k+1] \times [0; 1]$
State $(q_i, C = m, D = n)$	$x = i + 2^{-m}, y = 2^{-n}$
$q_1 : D{+}{+};$ goto q_2	$\begin{cases} x := x+1 \\ y := y/2 \end{cases}$ if $1 < x \leq 2$
$q_2 : C{-}{-};$ goto q_3	$\begin{cases} x := 2(x-2)+3 \\ y := y \end{cases}$ if $2 < x \leq 3$
q_3 : if $C > 0$ then goto q_2 else q_1	$\begin{cases} x := x-1 \\ y := y \end{cases}$ if $3 < x < 4$ $\begin{cases} x := x-2 \\ y := y \end{cases}$ if $x = 4$

MM: (q_i,0,0)…->(q_j,*,*)

ssi PAM: (i+1,1) …->le carré j<x<=j+1

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. . . finally we have proved:

Theorem (Koiran et al.)

Reach *is undecidable for 2d PAMs.*

Conclusions of Day 1

We have learned today

- What is a Hybrid Automaton.
- How to read yet another definition of HA and its semantics.
- How to model things using HA.
- Famous classes of HA.
- Safety verification as reachability problem.
- How to prove undecidability by simulation of Minsky Machines.
- Even the simplest classes of HA have undecidable reachability.

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verification

Abstract algorithm - important

A generic verification algorithm A

Forward breadth-first search

F=Init
repeat
    F=F $\cup$ SuccFlow(F) $\cup$ SuccJump(F)
until (F$\cap$ Bad $\neq \emptyset$)| fixpoint | tired
say "reachable" | "unreachable" | "timeout"

Most verification methods and tools are variants of it.

Abstract algorithm - important

A generic verification *semi*-algorithm A

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A generic verification semi-algorithm A

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There are variants:

- forward/backward
- breadth first/depth first/best first/etc.

Most verification methods and tools are variants of it.

How to implement it

Needed data structure for representation of subsets of $\mathbb{R}^n$, and algorithms for efficient computing of

- unions, intersections;
- inclusion tests;
- SuccFlow;
- SuccJump.

How to implement it

Needed data structure for representation of subsets of $\mathbb{R}^n$, and algorithms for efficient computing of

- unions, intersections;
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It could be *exact* or *over-approximate*.

Some trivial results

Theorem

If for a class of HA the Algorithm A can be implemented (exactly), then

- Reach *is semi-decidable;*
- *bounded* Reach *in n steps is decidable;*
- *a verification tool can be built.*

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Fact

Suppose for a class of HA the Algorithm A can be implemented approximately. Then we can build a verification tool saying:

- *"Unreachable".*
- *"Maybe reachable".*
- *" Timeout".*

TA: an interesting subclass of HA

Decidability

Automata and language theory

Verification of TA in practice

Part II

Timed Automata

Outline

3 TA: an interesting subclass of HA

4 Decidability

5 Automata and language theory

6 Verification of TA in practice

Outline

3 TA: an interesting subclass of HA

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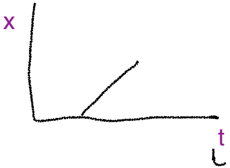
Definition of TA

Definition

Timed automata are a subclass of hybrid automata:

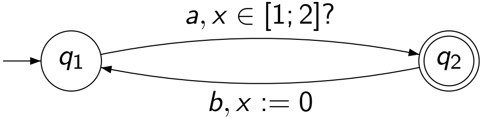
- **Variables** $x_1, \dots, x_n$, called clocks.
- **Dynamics** $\dot{x}_i = 1$, for all clocks, in all locations.
- **Guards and invariants** Conjunctions of $x_i < c$ (or $\leq, =, . \geq$))with $c \in \mathbb{N}$
- **Resets** $x_i := 0$ for some clocks.



x= temps écoulé après le dernier reset

An example of a timed automaton

- Timed automaton (we forget to write $\dot{x} = 1$):



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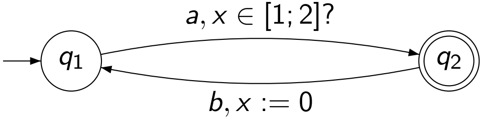

- Its run

$$(q_1, 0) \xrightarrow{1.83} (q_1, 1.83) \xrightarrow{a} (q_2, 1.83) \xrightarrow{4.1} (q_2, 5.93) \xrightarrow{b} (q_1, 0) \xrightarrow{1} (q_1, 1)$$

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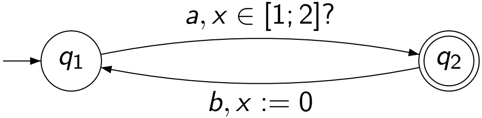

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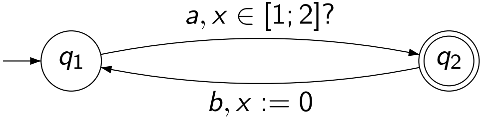

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- Its *trace* $1.83\, a\, 4.1\, b\, 1\, a$ a *timed word*
- Its *timed language*: set of all the traces starting in q_1, ending in q_2:

$$\{t_1\, a\, s_1\, b\, t_2\, a\, s_2\, b \ldots t_n\, a \mid \forall i . t_i \in [1;2]\}$$

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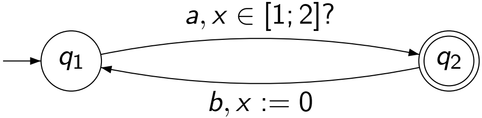

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Observation

Clock value of x: time since the last reset of x.

Some simple exercises

Draw timed automata for specifications:

- Request a arrives every 5 minutes.

Some simple exercises

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- Request a arrives every 5 minutes.
- Request a arrives every 5 to 7 minutes.

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- a arrives every 5 to 7 minutes; and b arrives every 3 to 10 minutes.
- Request a is serviced within 2 minutes by c or rejected within 1 minute by r.
- ~~The same, but a arrives every 5 to 7 minutes.~~

Meditation on TA

Compared to HA

Very restricted: only time progress remains from all physics.

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Compared to finite automata

Time and events together. Interesting ......

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For timed protocols, scheduling, timed aspects of embedded/real-time software (non-functional). See scheduling exercise.

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As specification formalism

For timed non-functional specifications. See exercises just above.

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3 TA: an interesting subclass of HA

4 Decidability

5 Automata and language theory

6 Verification of TA in practice

Main theorem

Theorem (Alur, Dill)

Reachability is decidable for timed automata.

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Unimed language of a timed automaton is regular (and can be computed).

Theorem (Alur, Dill)

Reachability is decidable for timed automata.

Classical formulation

Empty language problem is decidable for TA

Proof idea

- Split the state space $Q \times \mathbb{R}^n$ into regions s.t.
  - all the states in one region have the same behavior;
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  use it to recognize the untimed language

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- What does it mean: the same behavior?
- How to invent it?

Proof idea

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  use it to recognize the untimed language

Two difficulties

- What does it mean: the same behavior? Bisimulation.
- How to invent it? A&D invented it using ideas of Berthomieu (Time Petri nets). In fact it is rather natural.

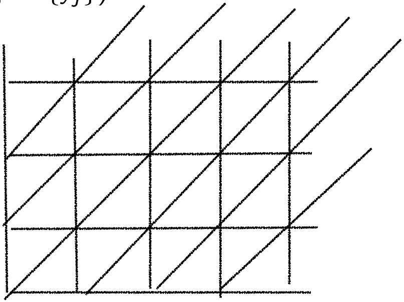
Region equivalence

Definition

Two states of a TA are region equivalent: $(q, \mathbf{x}) \approx (p, \mathbf{y})$ if

- Same location: $p = q$
- Same integer parts of clocks: $\forall i \, (\lfloor x_i \rfloor = \lfloor y_i \rfloor)$
- Same order of fractional parts of clocks $\forall i, j \, (\{x_i\} < \{x_j\} \Leftrightarrow \{y_i\} < \{y_j\})$

Look at the picture!



<=> x and y satisfy the same constraints of forms x_3<5 and x_1-x_2<2

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Look at the picture!

An issue

- Infinitely many equivalence classes.

Region equivalence

Definition

Two states of a TA are region equivalent: $(q, \mathbf{x}) \approx (p, \mathbf{y})$ if

- Same location: $p = q$
- Same integer parts of small clocks: $\forall \text{small} i \,(\lfloor x_i \rfloor = \lfloor y_i \rfloor)$
- Same order of fractional parts small of clocks $\forall \text{small} i, j \,(\{x_i\} < \{x_j\} \Leftrightarrow \{y_i\} < \{y_j\})$
- Or they are both big : $\forall i \,((x_i > M) \Leftrightarrow (y_i > M))$

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An issue, and a solution

finitely many equivalence classes.

- Solution: when a variable is BIG, we don't care about it.

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Definition

Equivalence classes of $\approx$ are called regions.

Region equivalence is a bisimulation

very informal

Equivalent states can make the same transitions, and arrive to equivalent states.

Region equivalence is a bisimulation

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Equivalent states can make the same transitions, and arrive to equivalent states.

Let us formalize it:

Lemma (time-abstract bisimulation)

Suppose $(q, \mathbf{x}) \approx (p, \mathbf{y})$. *Then*

Jump *If* $(q, \mathbf{x}) \xrightarrow{a} (q', \mathbf{x}')$ *then* $(p, \mathbf{y}) \xrightarrow{a} (p', \mathbf{y}')$ *with* $(q', \mathbf{x}') \approx (p', \mathbf{y}')$.

Time *If* $(q, \mathbf{x}) \xrightarrow{t} (q', \mathbf{x}')$ *then* $(p, \mathbf{y}) \xrightarrow{\hat{t}} (p', \mathbf{y}')$ *with* $(q', \mathbf{x}') \approx (p', \mathbf{y}')$ *(the time can be different!)*.

Reading a timed word

Iterating the previous lemma we get

Lemma

Suppose $(q, \mathbf{x}) \approx (p, \mathbf{y})$*, and* $q \xrightarrow{w} (q', \mathbf{x}')$ *(with some timed word* w*), then* $(p, \mathbf{y}) \xrightarrow{\hat{w}} (p', \mathbf{y}')$ *with* $(q', \mathbf{x}') \approx (p', \mathbf{y}')$ *(the timing in* $\hat{w}$ *can be different from* w*).*

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The untiming is the same

Corollary

The same set of regions is reachable from elements of one region. **(using the same untiming)**

Untiming ~~Decision~~ algorithm

- Build a region automaton RA
  - States are regions.
  - There is a transition $r_1 \xrightarrow{a} r_2$ if some (all) element of r_1 can go to some element of r_2 on a.
  - There is a transition $r_1 \xrightarrow{\tau} r_2$ if some (all) element of r_1 can go to some element of r_2 on some $t > 0$

(τ should be ε)

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  - There is a transition $r_1 \xrightarrow{\tau} r_2$ if some (all) element of r_1 can go to some element of r_2 on some $t > 0$
- ~~Check whether some final region in RA is reachable from initial region.~~

RA recognizes the untiming of the initial language

initial states of RA: regions of (i,0) for initial i of TA

final states of RA: regions of (f,x) for final f of TA, and any x

Outline

3 TA: an interesting subclass of HA

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Closure property

Definition

Timed regular language is a language accepted by a TA

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Theorem

Timed regular languages are closed under $\cap, \cup$, *projection, but not complementation.*

Closure property

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Theorem

Timed regular languages are closed under $\cap, \cup$*, projection, but not complementation.*

Fact

Determinization impossible for timed automata.

Decidability properties

Definition

Timed regular language (TRL) is a language accepted by a TA

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Proof.

Immediate from Alur&Dill's theorem. $\square$

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Proof.

Encoding of runs of Minsky Machine as a timed languages. $\square$

Reminder: regular expressions

Definition

Regular expressions: $E ::= 0 \mid \varepsilon \mid a \mid E + E \mid E \cdot E \mid E^*$

Theorem (Kleene)

Finite automata and regular expression define the same class of languages.

Reminder: regular expressions

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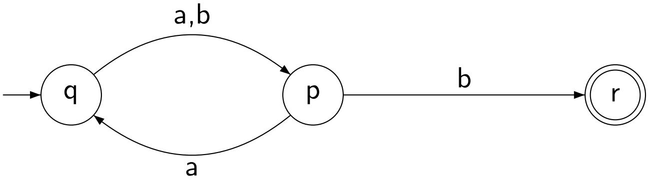
Example



$$((a+b)a)^*(a+b)b$$

Timed regular expressions

A natural question

How to define regular expressions for timed languages?

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How to define regular expressions for timed languages?

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Semantics:

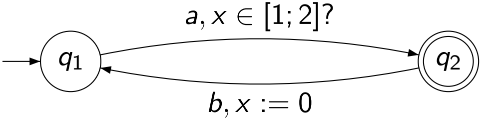
$$\|\underline{\mathbf{t}}\| = \mathbb{R}_{\geq 0} \quad \|a\| = \{a\} \qquad \|0\| = \emptyset \quad \|\varepsilon\| = \{\varepsilon\}$$

$$\|E_1 \cdot E_2\| = \|E_1\| \cdot \|E_2\| \qquad \|E_1 + E_2\| = \|E_1\| \cup \|E_2\|$$

$$\|\langle E \rangle\|_I = \{\sigma \in \|E\| \mid \ell(\sigma) \in I\} \qquad \|E^*\| = \|E\|^*$$

$$\|E_1 \wedge E_2\| = \|E_1\| \cap \|E_2\| \qquad \|[a \mapsto z]E\| = [a \mapsto z]\|E\|$$

A good example and a theorem



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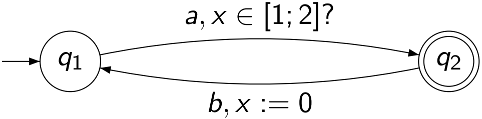


$$\{L = \{t_1\, a\, s_1\, b\, t_2\, a\, s_2\, b \dots t_n\, a \mid \forall i. t_i \in [1;2]\}$$

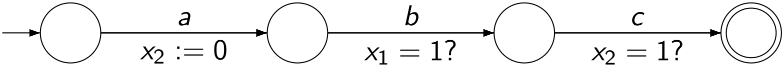
An expression for L : $\left(\langle \underline{\mathbf{t}} a\rangle_{[1;2]} \underline{\mathbf{t}} b\right)^*$

Theorem (A., Caspi, Maler)

Timed Automata and Timed regular expressions (with $\wedge$ and $[a \mapsto z]$) define the same class of timed languages

A nasty example

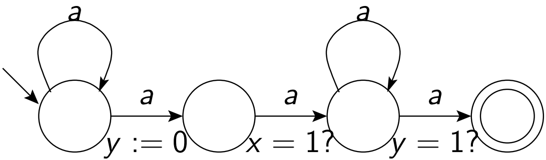
Intersection needed [ACM]



$$\{t_1 a t_2 b t_3 c \mid t_1 + t_2 = 1, t_2 + t_3 = 1\} = \underline{\mathbf{t}} a \langle \underline{\mathbf{t}} b \underline{\mathbf{t}} c \rangle_1 \wedge \langle \underline{\mathbf{t}} a \underline{\mathbf{t}} b \rangle_1 \underline{\mathbf{t}} c$$

Another nasty example

Renaming needed [Herrmann]



$$[b \mapsto a]\Big((\underline{\mathbf{t}}a)^*\langle \underline{\mathbf{t}}b(\underline{\mathbf{t}}a)^*\rangle_1 \wedge \langle (\underline{\mathbf{t}}a)^*\underline{\mathbf{t}}b\rangle_1(\underline{\mathbf{t}}a)^*\Big).$$

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Model-checking etc.

Reminder: decidability for TA

PSPACE-complete

- We can decide: Reach, $L \neq \emptyset$, $L \cap M = \emptyset$, $w \in L$
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Verification problem

Given a system S and a property P, verify that S satisfies P.

Verification approaches

For simple safety properties:

- Represent S by a TA A_S.
- Represent P as ¬Reach(Init,Bad). Empty language
- Apply reachability algorithm.(empty language)

For all kind of properties

(even with ω-behaviors)

- Represent S by a TA A_S. language = possible behaviors
- Represent $\neg P$ by a TA $A_{\neg P}$. language=bad behaviors
- Check that $L(A_S) \cap L(A_{\neg P}) = \emptyset$

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- Check that $L(A_S) \cap L(A_{\neg P}) = \emptyset$

Or express P in a temporal logic and use some model-checking.

A simple verification example

Exercise

How to verify this?

System A bus passes every 7 to 9 minutes. A taxi passes every 6 to 8 minutes. At noon a bus and a taxi passed.

Property Between 12:05 and 12:30, within 5 minutes after every bus, a taxi passes.

Reachability in practice: no regions

Fact

Real verification tools, e.g. UPPAAL, *do not use the region automaton. They apply a variant of the algorithm we know.*

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Algorithm B

F=Init
repeat
    F=F $\cup$ SuccFlow(F) $\cup$ SuccJump(F)
    Widen(F)
until (F$\cap$ Final $\neq \emptyset$)| fixpoint
say "reachable" | "unreachable"

Zones and DBMs

What is needed to implement Algorithm B

Data structure and basic algorithms for subsets of $Q \times \mathbb{R}^n$

Zones and DBMs

What is needed to implement Algorithm B

Data structure and basic algorithms for subsets of $Q \times \mathbb{R}^n$

Definition

Let $x_0 = 0$; let $x_1, \ldots, x_n$ - clocks.

- *Zone:* polyhedron defined by a conjunction of constraints $x_i - x_j \leq d_{ij}$ (or $<$) wirh $d_{IJ} \in \mathbb{N}$.
- *Difference bound matrix (DBM) for a zone:* $D = (d_{ij})$.

Fact

A zone is a union of regions.

Zones and verification of TA

Fact

Using DBMs, the following tests and operations on zones are easy ($O(n) - O(n^3)$):

- $Z_1 = Z_2$?; $Z = \emptyset$?; $Z_1 \cap Z_2$.
- $SuccFlow(Z)$ *and* $Succ_\delta(Z)$ *- both are zones.*

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See Cormen, graph algorithms.

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- $SuccFlow(Z)$ *and* $Succ_\delta(Z)$ *- both are zones.*

Corollary

Unions of zones, represented $(q_1, D_1), \ldots (q_n, D_n)$*, are suitable to implement Algorithm B*

Termination

Algorithm B

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say "reachable" | "unreachable"

Termination

Algorithm B

F=Init
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F=F $\cup$ SuccFlow(F) $\cup$ SuccJump(F)
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To ensure termination we must widen
In each DBM, when $c_{ij} > M$ replace $c_{ij} := \infty$.

When C_ij <-M replace c_ij:=-M

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Theorem

Algorithm B is correct and terminates (and used in practice)

Part III

Back to Hybrid automata: decidability

Outline

7. Decision by reduction to TA

8. Decision using finite bisimulations

9. Decision using planar topology

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7 Decision by reduction to TA

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Reduction to TA : simple cases

Fact

Reachability is decidable for the following subclasses of HA, it is reduced to TA reachability.

- *Like TA, rational constants.*

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Reachability is decidable for the following subclasses of HA, it is reduced to TA reachability.

- *Like TA, rational constants.*
  Reduction: *Multiply all the guards by the common denominator K, you obtain a timed automaton with the same reachability (location to location).*

Reduction to TA : simple cases

Fact

Reachability is decidable for the following subclasses of HA, it is reduced to TA reachability.

- *Like TA, rational constants.*
- *Like TA, but the rate of each clock = arbitrary rational:* $\dot{x}_i = r_i$ *(the same everywhere).*

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- *Like TA, rational constants.*
- *Like TA, but the rate of each clock = arbitrary rational: $\dot{x}_i = r_i$ (the same everywhere).*
  Reduction: *Change of variables $\bar{x}_i = x_i/r_i$ (and corresponding change guards) transform the system into a TA with the same reachability.*

Reduction to TA : simple cases

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- *Like TA, rational constants.*
- *Like TA, but the rate of each clock = arbitrary rational:* $\dot{x}_i = r_i$ *(the same everywhere).*
- **Initialized skewed-clock automata** *Like TA, but in a state* q *we have that* $\dot{x}_i = r_{iq}$ *(may depend on the state).* **Restriction:** *when we change rate, we forget the value. Formally, for any transition* $p \to q$*, either* $r_{ip} = r_{iq}$ *or* x_i *is reset.*

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  Restriction:*when we change rate, we forget the value.*
  Formally, for any transition $p \to q$*, either* $r_{ip} = r_{iq}$ *or* x_i *is reset.*
  Reduction*: Change of variables* $\bar{x}_i = x_i / r_{iq}$ *at state* q*. It works because of the restriction.*

Rectangular Hybrid Automata

Let us generalize

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Rectangular Hybrid Automata

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The class of Rectangular Hybrid automata is defined as follows:

- Variables $x_1, \ldots x_n$.
- Dynamics at each state q : inclusion $\dot{x}_i \in [a_{iq}, b_{iq}]$ (for each i)
- Invariant at each state q, and guard of each transition : $x_i \in [a_., b_.]$
- Reset on each transition : either x_i is unchanged, or it is set to an arbitrary point of some interval : $x_i :\in [a_., b_.]$.

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Fact

Reachability is undecidable for RHA.

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To obtain reachability one needs a restriction:

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Initialized RHA should reset x_i on each transition that changes its rate.

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Definition (When we change rate, we forget the value)

Initialized RHA should reset x_i on each transition that changes its rate.

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Reachability is decidable for Initialized RHA.

Probably the "largest" known decidable class of HA!

Outline

7 Decision by reduction to TA

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o-minimal automata

They have a complex, sometimes nonlinear dynamic, but they also forget the variable, when its equation changes.

Part IV

Conclusions and perspectives

Timed: Conclusions for a pragmatical user

- A useful and proper model of computer systems immersed in physical time : TA.
- Modeling and specification languages available.
- Efficient simulation, verification and synthesis tools available.

Timed: perspectives for a researcher

- Develop a theory of timed languages. Algebra, logic, topology etc. (see my text `http://hal.archives-ouvertes.fr/hal-00157685`)
- Improve verification techniques.
- Study rich and decidable specification formalisms (logical, algebraic, etc.) for timed languages.
- etc.

Quantitative verification
Information theory
Runtime verification/monitoring
Pattern-matching
Machine learning