MPRI 2-24-1: Algorithms and Uncertainty (2024)

Homework 3

Due on October 10, beginning of class

Instructions You can write your solutions either in English or French. Please observe the homework policy as described in the course web page.

Access graph model (5 marks) 1

We consider the paging problem with a cache of size k. For a given graph, where vertices are pages, if the last requested page is p, then the next page to be requested is a neighbor of p in the graph.

Show that the following algorithm has competitive ratio $O(\log k)$ in the access graph model on the cycle graph of length k + 1.

Algorithm FAR Whenever there is a page fault, do the following steps. If all pages in the cache are marked, then unmark all pages and start a new phase. Now the unmarked pages are consecutive on the cycle, i.e. form a path. Evict the median vertex of this path, breaking ties arbitrarily if any.

2 Markov model (5 marks)

We are given N pages and a cache of capacity k. If the last requested page was p, then the next page to be requested is q with probability $\alpha(p,q)$. Given N, k, α , design a linear program in $N\binom{N}{k}$ variables, which describes the optimal paging

policy.