## MPRI 2-24-1: Algorithms and Uncertainty (2024)

Homework 2

Due on October 3, beginning of class

**Instructions** You can write your solutions either in English or French. Please observe the homework policy as described in the course web page.

## 1 Handbags (5 marks)

A fashionista produces handbags of various sizes. To keep the problem simple, it takes her  $t \in \mathbb{R}$  time units to produce a handbag of size t, and she can sell it for t Euros. She cannot sell unfinished bags though. She starts to work at time 0, and at some moment  $T \ge 1$ , which is unknown to her in advance, a rich buyer enters her shop and buys the biggest available bag. If she sells a bag of size t, we say that her competitive ratio is T/t. Now she wants to minimize this ratio and asks you for help.

Prove that this problem is equivalent to the online bidding problem seen in class, in the sense that every optimal deterministic algorithm for this problem translates into an optimal deterministic algorithm for online bidding and vice versa.

## 2 Restricted bidding (5 marks)

For the restricted online bidding problem, we are given a set  $U \subseteq \mathbb{R}$ , which is closed, satisfies  $\min U \ge 1$  and might even be finite. Both the unknown threshold and bids have to belong to U. Show that the following bidding sequence is 4-competitive, namely  $B = \{\max\{U \cap [0, 2^i]\} : i \in \mathbb{N}\}$ .